Hierarchical Tree-Structured Stick-Breaking Priors

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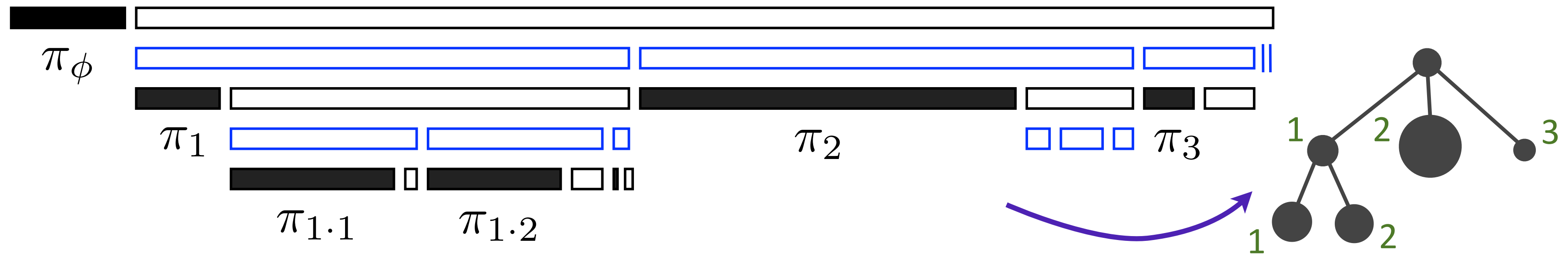
Overview

- Current models ignore the relationships between hidden states
  - e.g., the states of HMM or PCFG are exclusive
- Propose the general nonparametric prior which induces *the latent hierarchy between the hidden states*
- Construct a *HMM on a tree*, and a tree-structured topic model
- Topic model works, but the HMM currently fails ⇒ why?

Tree-Structured Stick-Breaking [1]

Partitions a unit interval hierarchically to get a measure on a tree

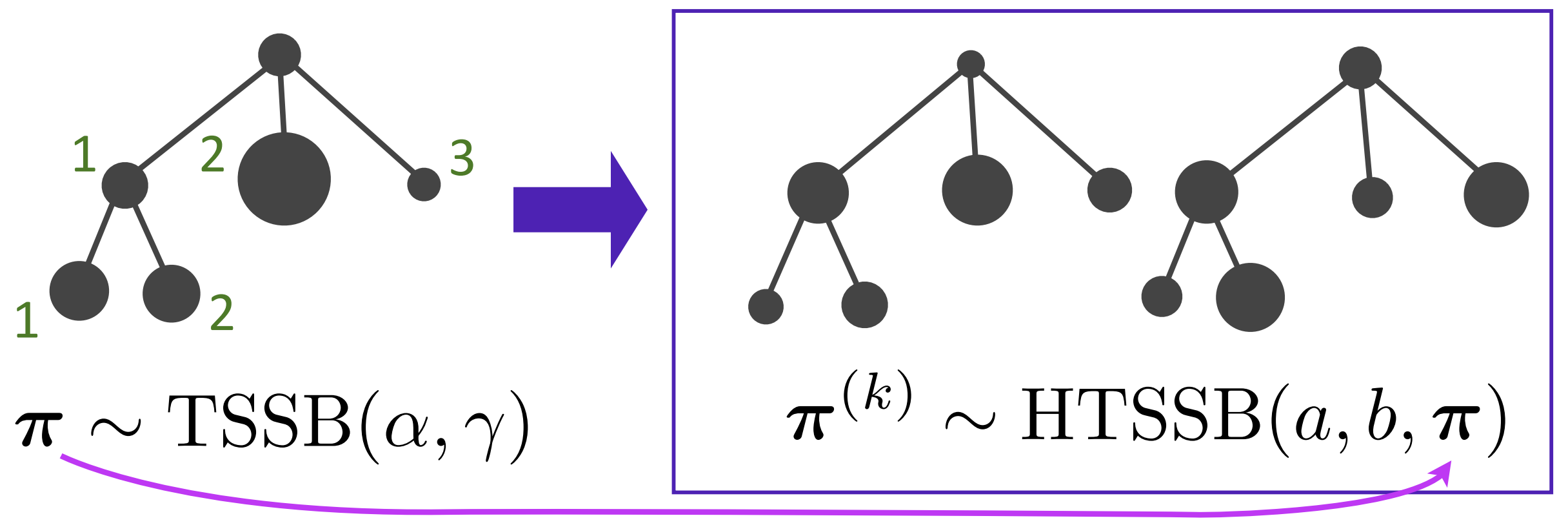


$$\boldsymbol{\pi} \sim \mathrm{TSSB}(\alpha, \gamma)$$

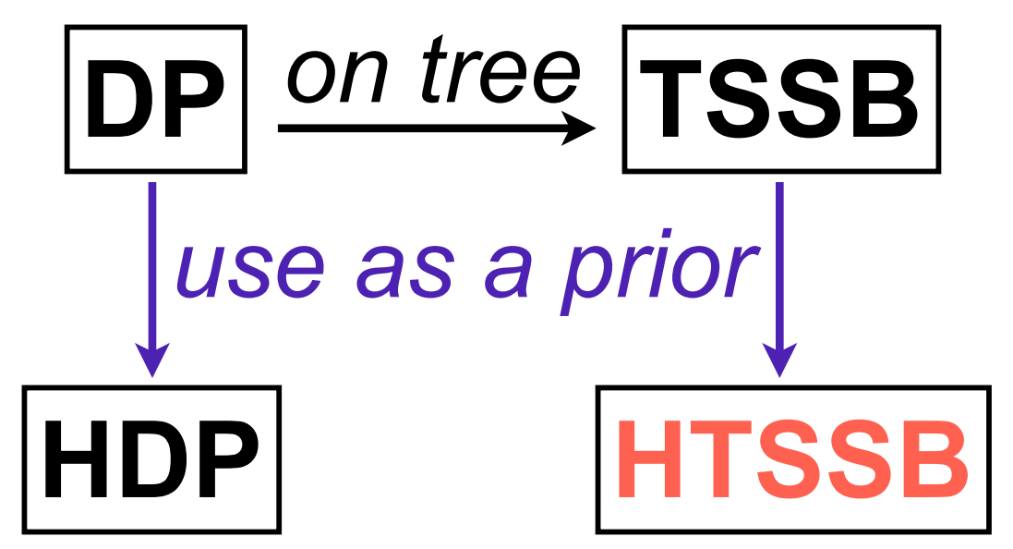
- The model consists of two kinds of stick-breakings:
  - ν-break selects stop or pass at the node: $\nu_\epsilon \sim \mathrm{Beta}(1, \alpha)$
  - ψ-break selects the child direction: $\psi_\epsilon \sim \mathrm{Beta}(1, \gamma)$
- Example: $\pi_{1\cdot 2} = (1-\nu_\phi) \cdot \psi_1 \cdot (1-\nu_1) \cdot (1-\psi_{1\cdot 1}) \cdot \psi_{1\cdot 2} \cdot \nu_{1\cdot 2}$
- Generalization of the Dirichlet process on the tree
- **Problem:**
  - Each draws from this prior creates *a different tree structure*
  - The same problem when extending the Dirichlet process to the grouped data, e.g., HDP-HMM ⇒ *define another type of hierarchy!*

Hierarchical TSSB: core idea

Use a draw from the TSSB as a base measure of another draw


1
2
3
1
2
$\boldsymbol{\pi} \sim \mathrm{TSSB}(\alpha, \gamma)$
$\boldsymbol{\pi}^{(k)} \sim \mathrm{HTSSB}(a, b, \boldsymbol{\pi})$

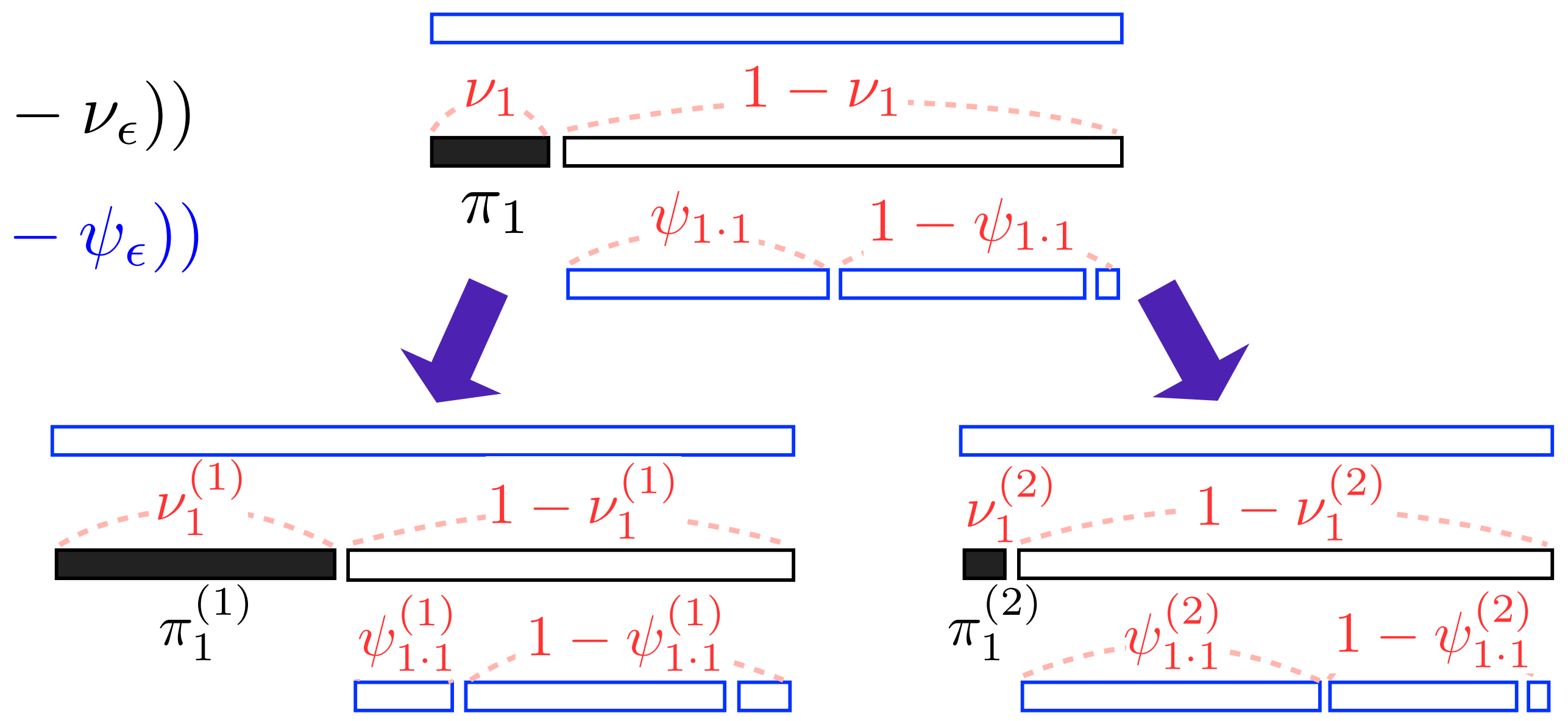
Model relationships:



Hierarchical ν- and ψ-breaks

Stick lengths of base measure are used as a prior in each position

$$\nu_\epsilon^{(k)} \sim \mathrm{Beta}(a\nu_\epsilon, a(1-\nu_\epsilon))$$
$$\psi_\epsilon^{(k)} \sim \mathrm{Beta}(b\psi_\epsilon, b(1-\psi_\epsilon))$$



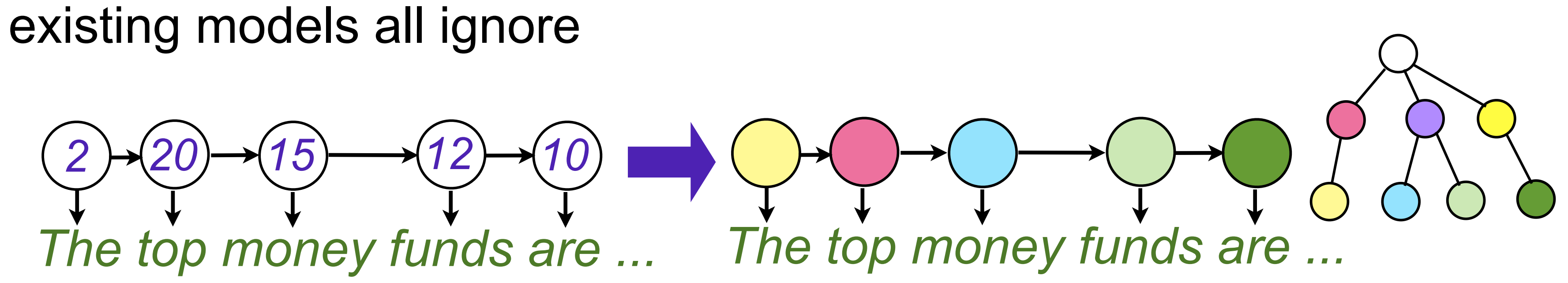
Difference between this ψ-breaks and the HDP

- Let $\varphi_{\epsilon i}^{(k)} = \psi_{\epsilon i}^{(k)} \prod_{j=1}^{i-1}(1-\psi_{\epsilon j}^{(k)})$ be the local branching prob. to i-th child
- If we model this branching process by HDP, $(\varphi_{\epsilon\cdot 1}^{(k)}, \varphi_{\epsilon\cdot 2}^{(k)}, \cdots) \sim \mathrm{Dir}(b\varphi_{\epsilon\cdot 1}, b\varphi_{\epsilon\cdot 2}, \cdots)$ holds, but in our model, it doesn't
- In HDP, the ψ-break is: $\psi_{\epsilon\cdot i}^{(k)} \sim \mathrm{Beta}(b\varphi_{\epsilon\cdot i}, b(1-\sum_{j=1}^{i}\varphi_{\epsilon\cdot j}))$
- Recently proposed nestedCRF [2] is based on HDP; ours is not

HMM on a Tree

Motivation: **We want to induce the latent hierarchy of states**

- In natural language processing, HMM or other probablistic grammar models are used to induce word categories for dimentionality reduction
- The word categories should comprise a hierarchical structure, which existing models all ignore



- Scientific question: How words are categorized in a tree?
- Engineering: the depth of the predicted state corresponds to the *confidence of that state*

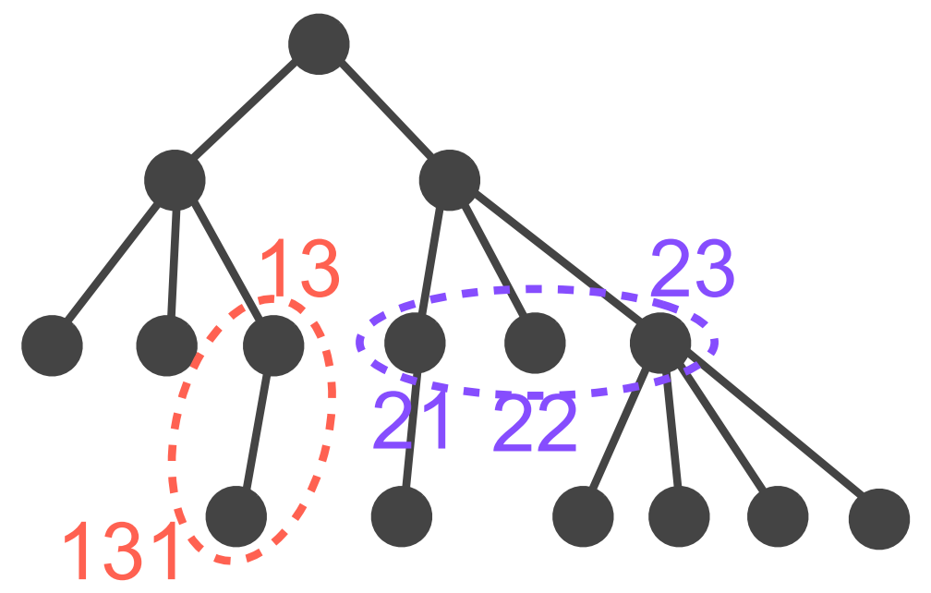
Assumption: **Two related categories are close to each other**

- Each node (category) has *a transition distribution* to other nodes

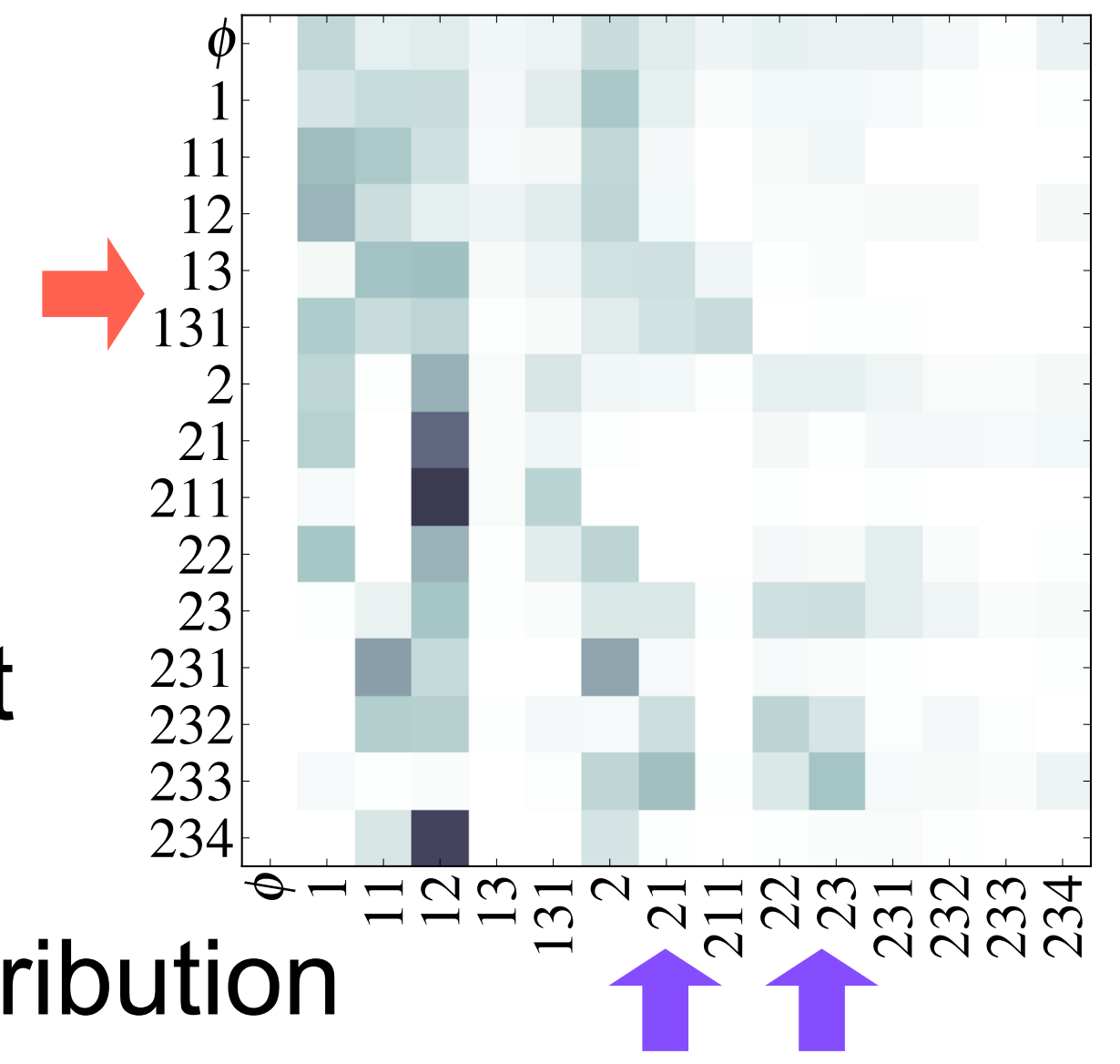
Generative process

1. Sample $\boldsymbol{\pi} \sim \mathrm{TSSB}(\alpha, \gamma)$ to define the global tree structure
2. On each node $\epsilon \cdot i$, sample $\boldsymbol{\pi}^{(\epsilon\cdot i)} \sim \mathrm{HTSSB}(a, b, \boldsymbol{\pi}^{(\epsilon)})$

A draw from the HMM prior



(i,j) : transition prob. from i to j



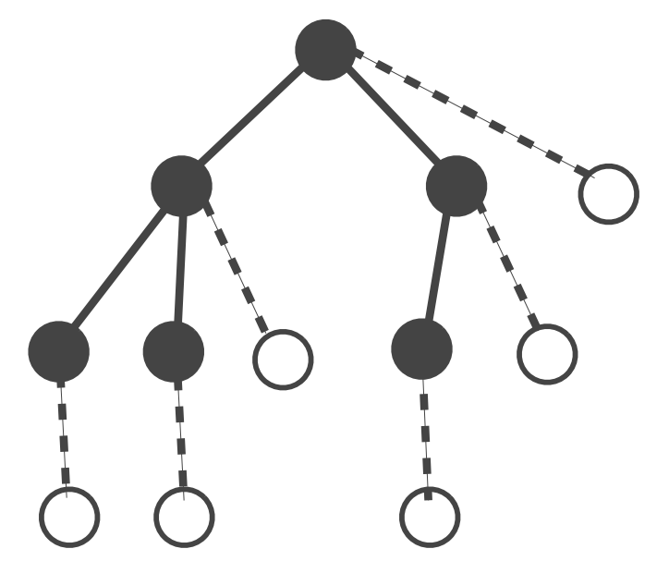
- $\boldsymbol{\pi}^{(13)}$ and $\boldsymbol{\pi}^{(131)}$ look similar:
  - Child node behaves similarly to the parent
- $\pi_{21}^{(\epsilon)}, \pi_{22}^{(\epsilon)}$, and $\pi_{23}^{(\epsilon)}$ are similar in many ϵ:
  - *Positive correlations in atoms* **in** each distribution

Inference

Gibbs sampler similar to the HDP-HMM:

$$p(z_t|\mathbf{z}^{-t}, \mathbf{w}) \propto p(z_t|z_{t-1})p(w|z_t)p(z_{t+1}|z_t)$$

To grow the tree, we place dummy nodes (like the dummy state of HDP-HMM)

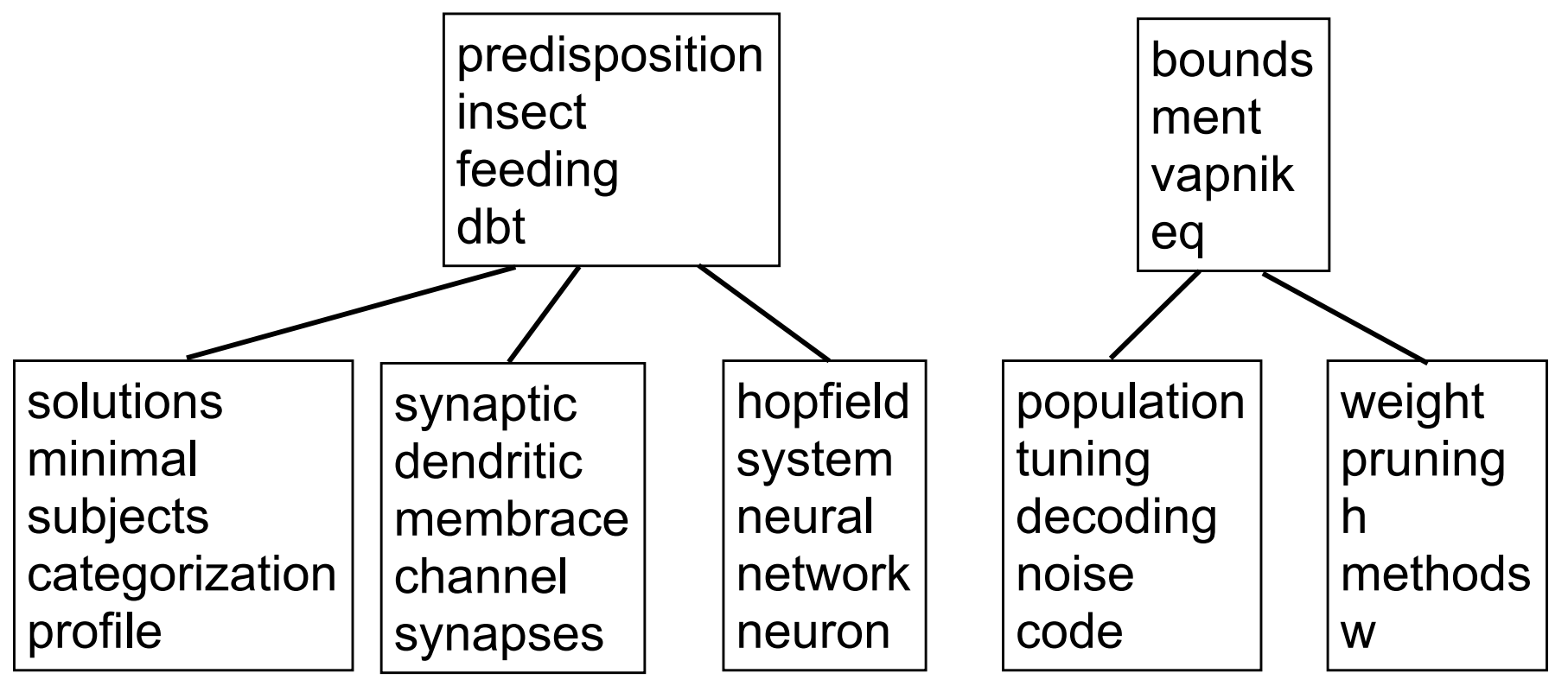


- Currently, this sampler for HMM doesn't work well
  - Similar word categories often appear in very different positions
  - because the effect of an ancestor diminishes in deeper nodes:

$$\boldsymbol{\pi}^{(\epsilon\cdot i)} \sim \mathrm{HTSSB}(a, b, \boldsymbol{\pi}^{(\epsilon)});\ \boldsymbol{\pi}^{(\epsilon\cdot i\cdot j)} \sim \mathrm{HTSSB}(a, b, \boldsymbol{\pi}^{(\epsilon\cdot i)});\ \cdots$$

Tree-structured Topic Modeling: $\boldsymbol{\pi}^{(d)} \sim \mathrm{HTSSB}(a, b, \boldsymbol{\pi})$

- It is easier than the HMM, so we can check the correctness of the model and sampler
- From the NIPS corpus, we got reasonable subtrees ⇒



Discussion

- For HMM to work, we need to solve several problems:
  - A blocked sampler, which enable larger moves, might be required
  - Theoretical analysis of the behavior *with deeper hierarchy*
- Interesting applications of Tree-HMM in other domains?

Reference

[1] R.P. Adams, Z. Ghahramani, and Michael I Jordan. Tree-structured stick breaking for hierarchical data. In *NIPS* 2010

[2] Amr Ahmed, Liangjie Hong, and Alexander Smola. Nested chinese restaurant franchise process: Applications to user tracking and document modeling. *In Proc. of ICML* 2013